Designing a Class of Geometries - From Concept to Instantiation

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Designing a class of Geometries-From Concept to Instantiation

A thesis submitted

in Partial fulfilment of the requirements
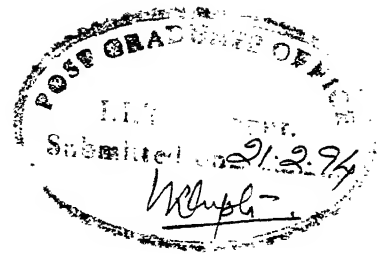
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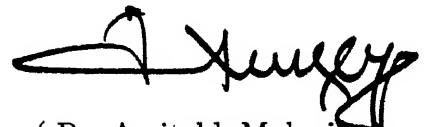# MASTER OF TECHNOLOGY

by

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to the

DEPARTMENT OF MECHANICAL ENGINEERING
INDIAN INSTITUTE OF TECHNOLOGY KANPUR
February, 1994

CERTIFICATE

It is certified that the work contained in this thesis entitled " *Designing a class of Geometries - From Concept to Instantiation*", by *Sarvesh Srivastava* has been carried out under my supervision and that this work has not been submitted elsewhere for a degree.

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ABSTRACT

Design proceeds in the direction of increasing constraint. In the conceptual stages, a few functional constraints evolve and define a basic shape that can be called a design template. By changing the parameters associated with this shape, the design can be made to fulfill different levels of functional demand. It is said that 80% of the design value is locked up in the initial design template. Today's CAD systems perform Drafting and Solid modeling tasks. Function modeling is beyond the capabilty of modern CAD systems. They can not represent a class instead they represent a single shape. They suffer from the weakness that a) they cannot model the abstract geometries that convey conceptual designs, and b) they do not have a mechanism for modeling the constraints that lead eventually to the final design.

The former makes it impossible to perform conceptual design, and the latter makes it difficult to edit the design later, when the basic functional constraints typically have to be regenerated. Recently, methods have been evolved for modeling the conceptual uncertainties using a *Hybrid Qualitative Spatial Model*, and providing visual feedback during the stage when the geometry is not well-defined [King, Josef Scott and Amitabh Mukerjee]. This tool can be used

- to perform Conceptual design with approximate shapes. At the end of this process, a design template as a basic shape is defined.

- obtaining a parametrization for the design template

- instantiating the parameters with a set of values for fulfilling specific level of functionality.

Thus, this is a vertically integrated work, starting from the ambiguous geometries that need to be modeled at the design phase, and ending with a fully instantiated design model. In this thesis, we extend this capability towards representing functions, by associating function related constraints with the parameters of the design template. These constraints are propagated by using Genetic Algorithm [Goldberg, David E.]. The design constraints and associated attributes are stored in a *Parameter Constraint Digraph*, which provides valuable information for any editing (change in some of the constraints) that may take place subsequently. PCD is very useful in selective alterations in design, while doing it manually. We use the design of a crankshaft to highlight the basic ideas behind the method, and show how differing initial templates lead to different popular crankshafts used in India today.

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Chapter 1

INTRODUCTION

Qualitative reasoning is a field that has been used to model complex design functionality, to represent spatial layouts, for process simulation, and other functions [1]. The degree of abstraction involved in qualitative reasoning is a function of the dimensionality of the space. While in one dimension, the abstraction of the real number line into the three regions (-, 0, +) involves an abstraction of the order of ∞^1, a similar abstraction in 3 dimensions is of the order of ∞^3. In many situations involving geometrical reasoning, this degree of abstraction is too coarse a grain, and something in between is needed. This is why, in the first part of this thesis, we use a hybrid qualitative model to represent the spatial constraints that emerge in the very first stages of conceptual design. The conceptual design model, by virtue of its qualitative nature, represents not a single geometric shape but an entire class. This is

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[1] see the special issue of *Artificial Intelligence*, October 1991, v.51, and particularly the model of clock function by Forbus et al
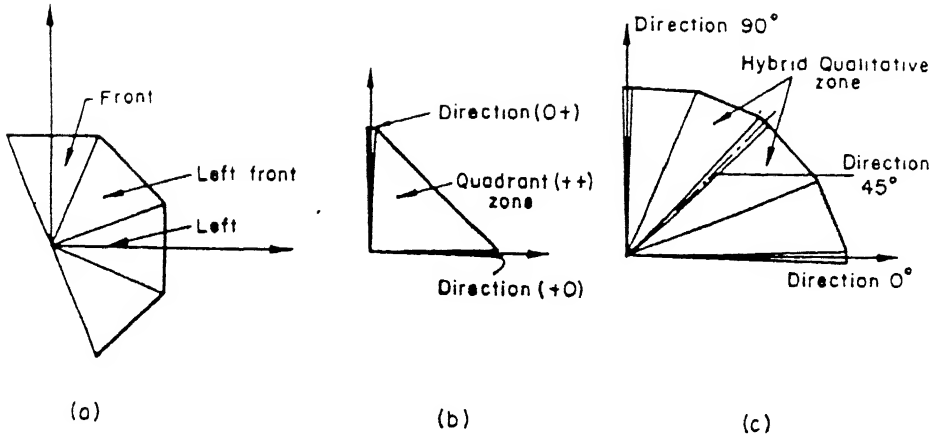
Figure 1.1: *Relative resolutions for the various angular zones.* In the hybrid model (c), The regions near the angles of 0 deg., 45 deg., 90 deg. are more precise than the other regions.

contrary to the unambiguousness criteria followed by traditional CAD geometric modelers [Requicha 90]. Indeed, one of the primary objectives of this work is to be able to build geometric models that *can* handle ambiguous definitions.

The basic idea is to represent a 2D contour, which is then swept along a 3D line, both the contour and the line being represented in the hybrid qualitative framework (Figure 1.2).A characteristic of this model is that the discretization used is not uniform - thus, angles close to zero or ninety degrees are represented fairly accurately while the intermediate angles are represented as zones. This is in contrast to purely qualitative models such as [Mukerjee and Joe 90] or [Hernandez 1992] (figure 1.1). Similarly, length ratios close to one are represented more accurately than other values.

This model deliberately creates objects that are ambiguous, i.e. each model

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POLYGON    CLEAR    Grid Function:  ⟳ OFF  Snap Function:  ⟳ OFF
EDIT       DONE     Grid Size   :  [50] 10 ▓▓▓▓▓▓▓▓▓▓▓ : 50
MAT        FILE I/O Position    :  (530,882)
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(a)   (b)
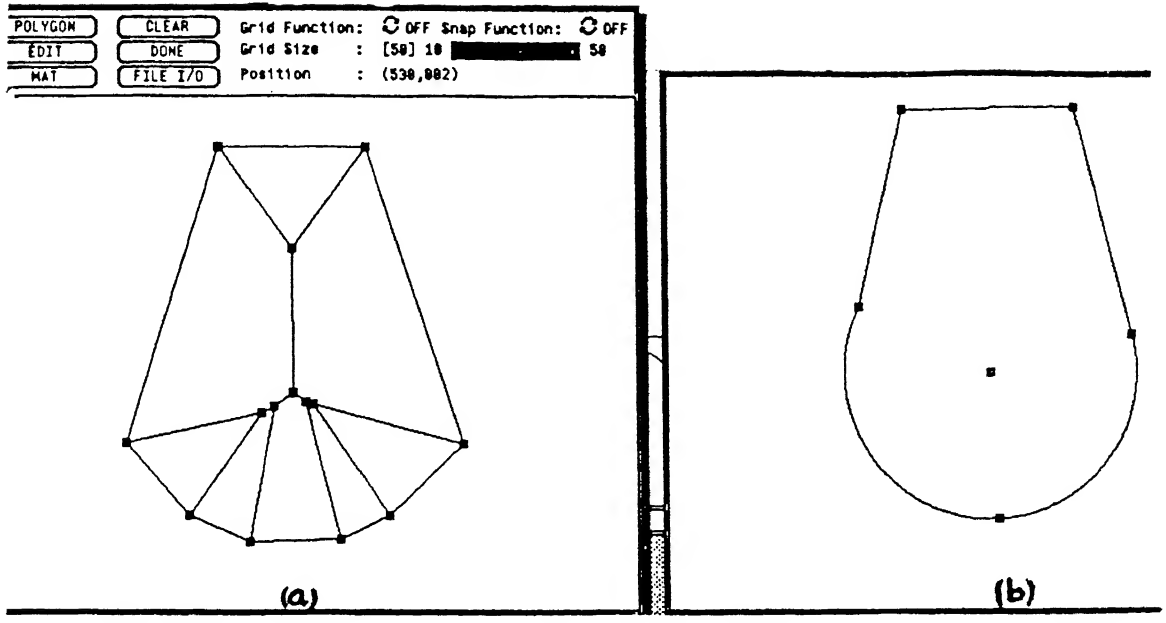
Figure 1.2: *Conceptual design of a crank contour.* Figure (a) shows the Voronoi diagram for an exact input. The corresponding Hybrid Qualitative Medial Axis Model represents a class of geometries, including this one. Figure (b) shows the inexact visualization process where a sample contour from this class is displayed for the user. Note that in the reduced resolution model, the polygonal lower sides are drawn as a circle.

represents a region in the space of qualitative contour and axis definitions. By improving the resolution in which this space is modeled, one obtains progressively smaller classes, i.e. models that are less ambiguous. The elements of the representation are axial, and resolution refers to the minimal discriminated length ratios and angles. Throughout this refinement process, visual feedback is available to the user in the form of *any* solution instantiation from the currently feasible set. As the design progresses the feasible set becomes smaller, and the instantiation becomes closer to the desired shape. The constraints applied at this stage of the design typically reflects some functional need in the design, but do not take into account details such as magnitude of forces, expected service life, etc. At the end of this process, we have a shape layout or a design template. The exact sizes of several parameters
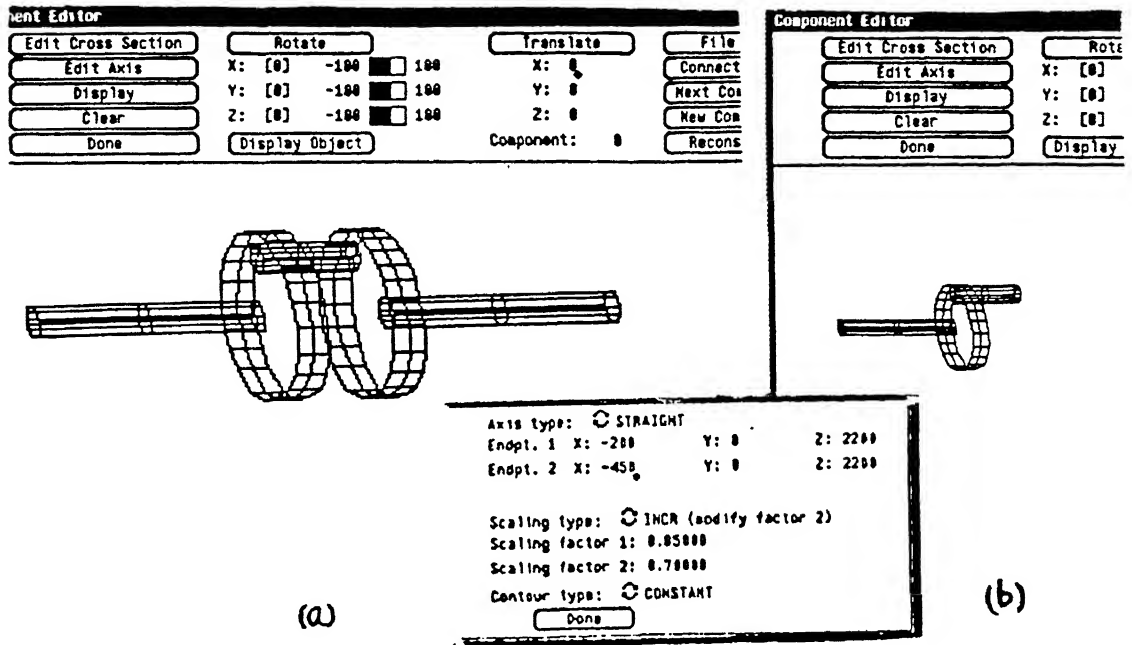
Figure 1.3: *Qualitative Generalized Cylinders for a circular cheek crankshaft.* Figure (a) shows the final design template for a circular contour crankshaft. Figure (b) shows an intermediate step in generating the 3D model, at which two of the components are not yet complete.

are unknown, but many of the attributes of this geometry are now linked by constraints that emerged during the conceptual design process. A visual reconstruction is also available using the hybrid qualitative reasoner. We call this model the parametrized design template, and this is the starting point for the second part of the design process, during which the constraints linking these parameters and various performance attributes are stated, and a directed graph is developed that links all the constraints according to their functional precedence. One of the key aspects of implementing such a model is the issue of managing constraints as the design is refined. The methodology proposed in this thesis is to organize these constraints in the form of a constraint graph, called *Parameter Constraint Digraph*. This results in a link between various aspects of the function of the part, and its geometries. The links in the graph point from more important constraints to lesser ones. Constraint equations are attached to features/parameters of the shape, and are eventually related to its dimensions. Subsequently, attempts to modify/add constraints to the system are tested against the existing constraint equations, and inconsistencies have to be explicitly overruled, which results in a restructuring of all dependent constraints (i.e. those constraints downstream from the modified one). The Parameter Constraint Digraph (*PCD*) provides a rich database for helping the user remain consistent with the basic constraints while editing the design during the many design revisions that typically follow an actual design. All functional attributes and inter-relations are also part of this design.

The next chapter looks at building geometric models for ambiguous objects. Section 3 presents the Parameter Constraint Digraph for organizing the functional constraints, and chapter 4 instantiates one such network with a crankshaft as an example. Section 5 points towards further work for integrating and expanding constraint representation for handling other objects.

Chapter 2

Models for ambiguous geometries:

There are two properties that a model for conceptual design must satisfy:

1. As the design precision increases, the shape must change in a continuous manner, and

2. the basic paradigm must be close to the human intuitive geometry definition process.

Based on this criteria, we can immediately rule out some of the standard geometric modeling techniques. Boundary representation models represent geometry in terms of surface attributes, which are positioned by defining some key points on the surface. In an ambiguous model, we could represent these point coordinates in the variable resolution model, but this would often

cause many such points to lie in the same resolution frame and would not be meaningful. At the same time, the number of such boundary points would themselves be a function of the precision and would not be suitable to such a treatment. Boolean constructs out of simple primitives (Constructive Solid Geometry) appears more promising to begin with, since the primitives are associated with certain parameters that can be defined with varying precision. However, the problem arises in the relative positions of the primitives, represented typically by a coordinate transformation. Such transformations do not behave well at low resolutions (e.g. if numbers can be distinguished only into the classes {-,0,+}, or represented by only four bits). Also, many of the constructions using CSG require one to define objects in a very convoluted and non-intuitive manner, which is not compatible with the objective (2) above. Similarly, decomposition methods such as octrees, though they provide good multi-resolution continuity, are not compatible with design intuition. Thus (2) tends to be a more binding constraint.

Human intuition of shape is often axis driven, as are almost all biological systems. Many constraints in engineering design depend on axis relations, such as that of parallelism between a crank-pin and the crank axis. A modeling paradigm that provides for such axis based design is *Generalized Cylinders*, in which a cross-section contour is swept along an axis, while possibly undergoing some continuous deformation. However, the problem with the generalized cylinder model is that the cross-section of the axis needs to be modeled, and it is not clear again how this can be done for a multi-resolution framework. If the contour is represented by its boundary, we face again the problem of

specifying the vertices in the low resolution modes. One possibility is that of chain coding, in which each vertex is specified as a relative displacement on the earlier one; by modeling these displacements in different resolutions one can obtain continuously deforming shapes. However, there were many problems at low resolution such as self-intersection and non-closure of the contour.

2.1 Fourier methods for representing 2D contours:

In this approach, one represents a contour using polar coordinates, where the (r,θ) relation is modeled as a Fourier transform. Variable precision is obtained by changing the number of terms in the function. Thus, the function is given by:

$$r = a_0 + a_1 sin\left(\theta + \psi_1\right) + a_2 sin(2\,\theta + \psi_2) + \cdots + a_n\, sin(n\,\theta + \psi_n)$$

where a_i represent the amplitude of the various fourier coefficients, and ψ_i represent the phase shifts.

Clearly, as n increases, the model becomes more precise, subject to the computational precision available. Figure 2.1 shows a variable resolution contour using fourier series. While Fourier Series seems to be a clean model for repre-
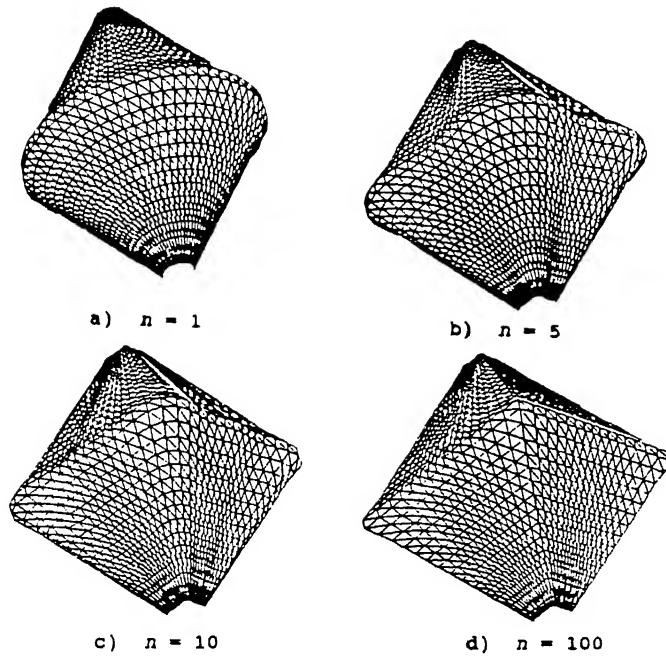
Figure 2.1: *Variable resolution using fourier series:* The object is an octahedron with sharp angles. By changing the number of terms in the Fourier Series representation, we can obtain varying levels of accuracy.

senting biological shapes, it is not well suited to conceptual design since the fourier coefficients cannot be obtained without first having a cross-section in hand. They also require highest resolution for contours with angular or nearly angular corners, a common enough situation in mechanical design. However, this approach holds a lot of promise for biomedical televisualization, for example.

2.2 Qualitative Medial Axis methods for 2D contours

Returning once again to the hypothesis that axes are important in the human thinking process, one could try to model the 2D contour itself using the axis approach. Such a model is variously known as the Medial Axis Transform [Blum] or the Line-Segment based Voronoi Diagram [Fortune 87] or the Grassfire Algorithm. The basic idea here is to model the shape by following the center of a maximally inscribed circle, which will then move along the bisectors of various edge segments. The contour thus developed represents a skeleton of the axial components. However, it is typically not amenable to multi-resolution analysis.

In this work, we represent the axial components using variable discretization of angle and length, a model that is an extension of a more general qualitative model of space [Mukerjee and Joe 90]. This model is a quantitative-qualitative hybrid, that starts off towards the qualitative end of the spectrum (low resolution), and moves towards more precision at the quantitative end. In figure 2.2 We trace the evolution of progressive detail in the cross-section of a knife. Had we used some other vertex based method for this cross-section, the similarities between the contours would have been lost, but using the medial axis model, the principal horizontal axis remains the same throughout.
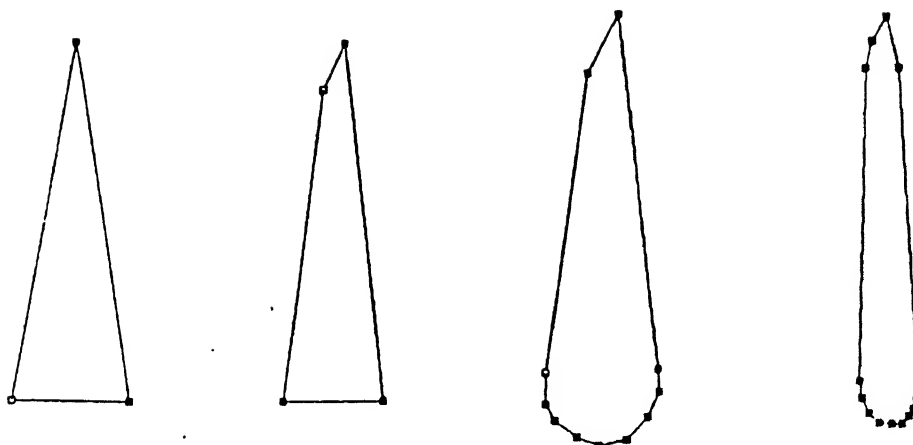
Figure 2.2: Progressive evolution of a knife cross section

2.3   Qualitative Generalized Cylinders

Generalized Cylinders are produced by translating a cross-section along an axis. In this case, both the axis and the cross-section are represented as a quantitative-qualitative hybrid, i.e. in some reduced resolution expressed in terms of the angular and linear resolutions. The axis can be any line expressible in the geometric model, but with the low resolution constraints it is usually linear. Another aspect of Generalized cylinders is the capability to vary the cross-section as one translates. This is simplified to one where the cross-section can increase, decrease, or remain constant. In constructing the hybrid models from the initial qualitative model, one has to make decisions regarding the level of resolution that is appropriate to the constraint level as required in the functional definition. This relationship is in no means easy to evolve. In addition to the mathematical aspects, there is also the cognitive aspect of the resolution i. e. the human designer must be able to

visualize the object from a graphic reconstruction with the low resolution model. This is considered in the next chapter. Subsequently, in chapter 3 and 4, we consider the issues for managing and organizing the constraints of design. By translating such a cross section along axes, 3D components are produced.

2.4 Concept Visualization: Displaying ambiguous models

In order for an ambiguous model to be useful as a design tool, the system must also provide the designer with *Visual Feedback*. Since there are many possible geometries corresponding to the design, and since the display can only model one or two such shapes, the question is that of deciding on which ones to display. It is found that in the initial phases of concept formation, the user is not too particular about many aspects of the shape. As the design progresses, the constraints cause increased precision, and it becomes more and more difficult to distinguish visually between the geometries being modeled. Thus, it appears that displaying *any* model of the shape will also provide the user with valuable feedback. Since finding any such instance implies solving the entire set of constraints, it is often computationally expensive to generate, especially since we must handle a wide class of nonlinear constraints. In order to model the interrelation between the constraints imposed by the user these are modeled in a hierarchy; a number of secondary functions depends on constraints that were imposed earlier. Thus, the newer constraints are

"lower" in the hierarchy, and any change of a "higher" constraint automatically negates the lower functionality. This creates a "Parameter Constraint Digraph" which can model the functional dependence of various parts of the design. A typical crankshaft is used to demonstrate this process.

2.5   Parametrization of the template

The designer starts conceiving shape which could satisfy basic functional requirements. Each time he draws some shape (represented using Qualitative model) new ideas appear in his mind. So he keeps on refining/modifying the shape till he is satisfied that the resulting shape is going to satisfy functional, manufacturing ease, aesthetic and other requirements. After *the* shape is fixed, we call this as the *template*. All the activities which are involved in reaching upto this template fall under the domain of conceptual stage. Depending upon the geometry/features of the template various parameters are defined so that the template geometry is fully expressed in terms of these parameters. We call this process of defining parameters as *parametrization*. All activities of design process which deal with a *particular* template fall under the domain of parametric design stage.

While evolving the shape of a crankshaft, many pre-parametric constraints need to be considered. For example, the shape must be able to act as an intermediate member between the connecting-rod and shaft and thereby transmit motion (oscillatory and linear) of connecting rod to rotary motion of the mainshaft. Some of the questions which arise in the designer's mind

(brain-storming) during conceptual stage are-1) Where should the crankpin be located. 2) Where should the big end of connecting rod hold the crankpin (inside or outside the cheeks). Again the alignment between mainshaft and crankpin needs to be kept in mind while generating/modifying shapes. The cheek's shape also bears many constraints. Functions to be performed by the cheek decide its shape. If it has to regulate the opening and closing of the transfer port; it should have a periphery of material that regulates the port for required angular displacement of the mainshaft. Many other requirements, e.g., aesthetic view, fillet radius at corners etc. are in designer's mind while he is giving visual representation (through Qualitative model) to his thoughts. Designer tries to evolve a shape which satisfies all these requirements.

Here the designer is mostly concerned with the shape of the product and not with the sizes. Once a particular shape is evolved (which may be of any size), depending upon external parameters (in case of a crankshaft-H.P., rpm, external load, bearing life.) on which the size/dimensions depend other sizes of the same template can be obtained. If we are designing a pump/turbine then dimensional analysis can be used to find different sizes.

2.6   Organizing the constraints

One point that needs to be clarified here is the difference between the constraints for initial shape definition and those for eventual parameter definition. Many times, the same conceptual template may fit more than one

object, or it may represent a specific class of object. a) *Constraints for Initial Shape Refinement:* Here the shape is built up from individual components. For each component, the cross-section is defined as a qualitative 2-D polygon. Constraints are also introduced for inter-component relations (e.g. in a knife the handle is parallel to the blade). b) *Parametric Constraints:* After the basic shape concept has been developed, a number of parameters are typically not specified. These can be instantiated with different parameter values to obtain different instances of the same object.

Chapter 3

Parameter Constraint Digraph

In any component, design dimensions are inter-related through constraint equations. These constraints can be represented in a directed graph (called *Parameter Constraint Digraph*, with the link directed from the higher to the lower priority constraint. Priorities are intended to represent the impact on overall functionality, if these can be clearly characterized. Otherwise, the links are bi-directional. With each constraint many attributes are associated. Some of these attributes are variables, tolerance, order, type (bound, inequality), nature (empirical, geometrical). All constraint equations are tagged with all its attributes. This information is used during constraint propagation (e.g. tolerances), or during the editing process, when certain constraints need to be overruled, modified, or replaced.

Constraints may be of various types. Geometric constraints like $L_c = l_1 + 2t + 2l_2$ are often exact, while others (e. g. empirical constraints) usually incorpo-

rate wide margins of safety, and can therefore be violated to some extent. Strength, thermal, or dynamic constraints would fall somewhere in between. To distinguish these different types of exactitudes during the constraint propagation process, we associate these equations with three tolerances - small, medium and large. These are written ϵ_s, ϵ_m, and ϵ_b, and correspond roughly to the three types of constraint outlined above. The exact values will depend on the design under consideration. We illustrate our approach using two examples - A simple 2-D engineering drawing with geometric constraints, [Light and Gossard], and a small engine crankshaft, differing designs of which are widely used in Indian scooters and mopeds. We also show how the constraints for the crankshaft can be propagated to obtain model that are very close to actual crankshaft designs, all belonging to the same class of design.

Example 1:

In one of the early and well known works on parametric representation of geometric contours, Light and Gossard [3] had presented a simple 2D part (figure 3.1). Since the functionality of this component is not known, there are no priorities and all the links are bidirectional. We have the following constraint equations.

$$X_2 - X_1 = C_1 \, , \, precision(\epsilon_s) \quad -1a$$

$$Y_2 - Y_1 = C_2 \, , \, precision(\epsilon_s) \quad -1b$$

$$Y_3 - Y_2 = C_3 \, , \, precision(\epsilon_s) \quad -1c$$

$$X_2 - X = C_4 \, , \, precision(\epsilon_s) \quad -1d$$

$$Y - Y_3 = R_3 \, , \, precision(\epsilon_s) \quad -1e$$

$$Y_4 - Y = C_5 \, , \, precision(\epsilon_s) \quad -1f$$
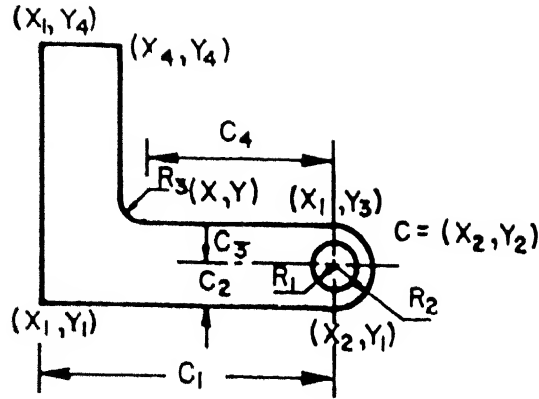
Figure 3.1: *A simple 2-D engineering drawing with geometric constraints* (from Light and Gossard).
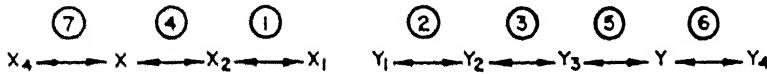
Figure 3.2: *Parameter Constraint Digraph.* when all variables are at equal ranking.

$$X_4 = X - R_3, \; precision(\epsilon_s) \quad -1g$$

The Parameter Constraint Digraph for this example is a linear graph as shown in figure 3.2.

Example 2:

A crankshaft is the part in an engine that converts the oscillating motion of a piston into rotating output motion. In smaller engines as used in scooters and small motorcycles, crankshafts of the type shown in figure 3.3 are common. Figure 1.3 shows how the design template for such a design is developed from

Figure 3.3: *Elementary model of scooter crankshaft.* This model is a design template with the parameters shown. Many other aspects of the design, e. g. parallelism of the main axis and the pin, have already been defined during the initial shape refinement.

ambiguous concepts. A number of parameters (size, lengths, etc) are open for further specification. The template itself defines a *parametric class*, and a broader *functional class* is formed by combining several such subclasses. The simplified model of such a crankshaft is shown in figure 3.3. Of the parameters shown, we focus on the central part of the crankshaft (i.e. within length L_c) since the rest of the shaft is used to mount the clutch and magneto which are usually external designs.

3.1 Constraint equations for an elementary crankshaft

This section models functional characterstics such as geometric continuity, strength in terms of constraint relations among the parameters of the crankshaft. In the following constraint equations, we use upper case symbols to indicate constants or external inputs, and lower case variables are the geometric design parameters of template (figure 3.3). Let us consider for example, the bearing selection problem, solving which involves a discrete search since bearings come in several discrete standard sizes. The consideration for bearing design is bearing life, L_i, revolutions which together with the radial load W_i defines the dynamic capacity $C_i = \frac{W_i L_i^{1/3}}{100}$. In the standard bearing catalogue, one can look up a bearing type with dynamic capacity greater than C_i. The dimensions of the bearing inner race determine the dimensions l_1 and d_1 in figure 3.3. We can write the constraint as

$$l_1, d_1 = f(C_1), \, precision(\epsilon_m) \tag{3.1}$$

where the function f() represents selection of bearing from catalogue, we use medium tolerance as we can choose a bearing one step above or below the optimum size, though suboptimal, our knowledge here is so approximate that it is unlikely to make a difference. Similarly for main bearing:

$$l_2, d_2 = f(Dynamic\,Capacity) = f(C_2) \tag{3.2}$$

Length equality constraint is given as

$$2t + l_1 + 2l_2 = L_c, \ precision(\epsilon_s) \tag{3.3}$$

where L_c is an external constant (axial length) for us as we are designing only crankshaft, but it will also become a variable if we are designing the whole engine and housing. Here precision is very small, the constraint being geometric. Crank-pin strength in bending gives the following

$$d_1 >= F_1 L_c^{1/3}, \ precision(\epsilon_m) \tag{3.4}$$

where F_1 is a constant. Since factor of safety is not exact so we can violate this constraint to some extent and hence the medium tolerance. Pin strength in radial crushing needs

$$S_{cw} l_1 d_1 >= F_2, \ precision(\epsilon_m) \tag{3.5}$$

where S_{cw} is the allowable crushing strength of the pin material. F_2 is the max. force on crank pin. Reason for employing medium tolerance is same as in constraint (3.4) above. Shaft strength in radial crushing requires

$$S_{cw} l_2 d_2 >= F_3, \ precision(\epsilon_m) \tag{3.6}$$

where F_3 is the max. force at the bearing. Reason for medium tolerance

$W_{21} + W_{22} = W_2$

θ_2 Varies from θ_C to θ_B

θ_2' Varies from θ_A to θ_B

θ_1 Varies between θ_{1A} to θ_{1B}

$2R_1 < r_b - \dfrac{d_2}{2}$

$r_b - 1.2 R_1 > r_1 > R_1 + \dfrac{d_2}{2}$

Figure 3.4: *Cheek with extended periphery for regulating transfer port.* Various parameters are shown.

application is same as in constraint (3.4). Cheek thickness is given as

$$t \epsilon \, (.22D - .32D), \, precision(\epsilon_b) \qquad (3.7)$$

where t is the check thickness and D is the cylinder dia. The tolerance is large since this is an empirical relation. Cheek cross width is given as

$$w >= \frac{F_4}{t}\left(1 + 3\frac{L_c - 2l_2 - t}{t}\right), \, precision(\epsilon_m) \qquad (3.8)$$

F_4 is a constant. Reason for employing medium tolerance is inexactness of factor of safety. Counterweight angle can be

$$\theta = 127 \, deg., \, precision(\epsilon_b) \qquad (3.9)$$

which is a traditional empirical value. Counterweight radius is

Constraint-no.	Variable	Tolerance	Order	Type	Nature.
1	l_1, d_1	ϵ_m	-	-	Standard.
2	l_2, d_2	ϵ_m	-	-	Standard.
3	t, l_1, l_2	ϵ_s	Linear	Equality	GEO
4	d_1	ϵ_m	Nonlinear	Inequality	SOM.
5	l_1, d_1	ϵ_m	Nonlinear	-	SOM.
6	l_2, d_2	ϵ_m	Nonlinear	-	SOM.
7	t	ϵ_b	linear	Bound	EMP.
8	w, t, l_2	ϵ_m	Nonlinear	Inequality	SOM.
9	θ	ϵ_b	Linear	Bound	GEO.
10	r_b, w, θ, r	ϵ_b	Nonlinear	Equality	Balancing.
11	r, d_1	ϵ_b	Linear	Equality	GEO.

Table 3.1: *Table shows dimensions affected by each constraint, tolerance associated with each constraint & their nature.* EMP.: empirical, SOM.: strength of material, Standard.: standardization., GEO.: Geometrical.

$$r_b = (\frac{1.5 \, w \, r^2}{\theta})^{1/3}, precision(\epsilon_b) \qquad (3.10)$$

r is the length of crank from crank axis and is subject to constraint

$$r = stroke/2. + d_1/2. + 8 \, mm, precision(\epsilon_b) \qquad (3.11)$$

Table 3.1 shows some characterstics of the constraint equations. This table which indicates the parameters inter-relations in the variable column, leads to the PCD of figure 3.5.

We have discussed above the constraints for an elementary crankshaft (figure

3.3). We have also shown how a table and thereby a PCD can be developed. Similarly differing PCDs are arrived at by considering different templates and new constraints:

In case of a circular cheek crankshaft (figure 4.5b), in strength criteria, we have one more equation, i.e.,

$$R = (r^2 + w^2/4)^{1/2}, \, precision(\epsilon_m) \tag{3.12}$$

Tolerance associated with the constraint is medium because w is decided on the basis of inexact factor of safety. PCD for circular crankshaft is shown in figure 3.6. In case of crank-cheek of figure 3.4, material saving and balancing govern parameters r_b and m

$$\frac{4}{3}r_b^3 t\rho = (M_r + M_c)\frac{s}{4} + \frac{\pi}{8}d_1^2 l_1\rho_1 s + \frac{2s}{3}m, \, precision(\epsilon_b) \tag{3.13}$$

Where M_r is the mass of reciprocating parts and M_c is the fraction of connecting rod mass at the crank-pin. ρ is the density of cheek material and ρ_1 is the density of the crank-pin material. s is the stroke length. m is the mass of upper half of each cheek. Here mass m is distributed in such a way so that center of mass for each upper cheek lies on the crank-arm axis at a height of $\frac{s}{3}$. The tolerance associated with the constraint is big as this can be over-ruled if any other relatively important constraint requires this. For this crankshaft, in strength criteria, we have one more equation, viz.,

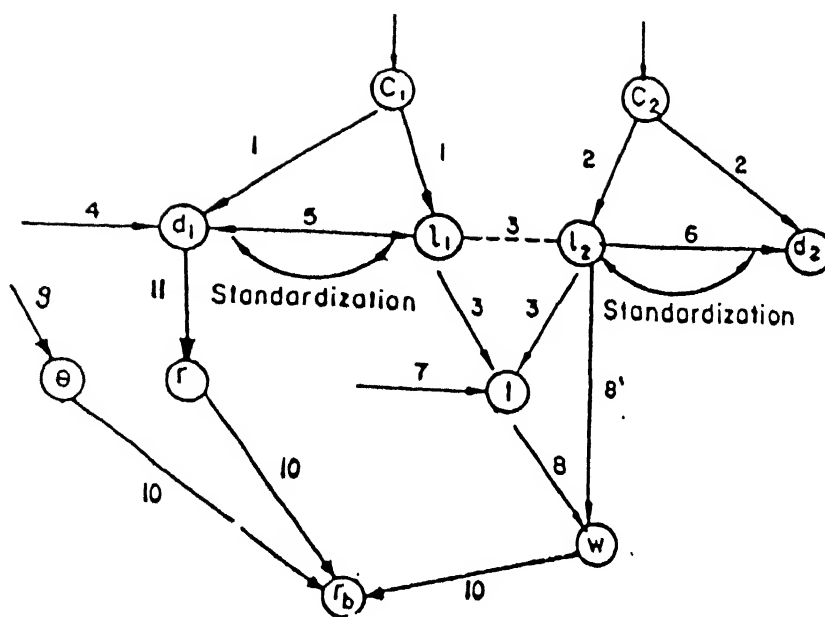$$w_1 + w_2 \geq w, \, precision(\epsilon_m) \tag{3.14}$$

Figure 3.5: *Parameter constraint digraph.* The nodes are dimensions, and the directed links between dimensions represent constraints. The link direction signifies the relative importance of the parameters. The dashed link without any direction shows that though the dimensions are connected, but in actual design they are treated independent of each other, i. e. they do not affect each other.

Here again medium tolerance is associated with the constraint as w is decided on the basis of inexact factor of safety. PCD for this case is shown in figure 3.7. This PCD can be further extended to take care of r_1, θ_1 R_1 etc.

The PCD nodes are the parameters of the design, which in this example are dimensions of the design template of figure 3.3. Priorities are assigned to variables in a subjective manner; the constraint propagation algorithm may or may not use these priorities. We followed these in our manual analysis (like most designers), but the Genetic algorithm based computational approach considers all design variables simultaneously and is priority independent.
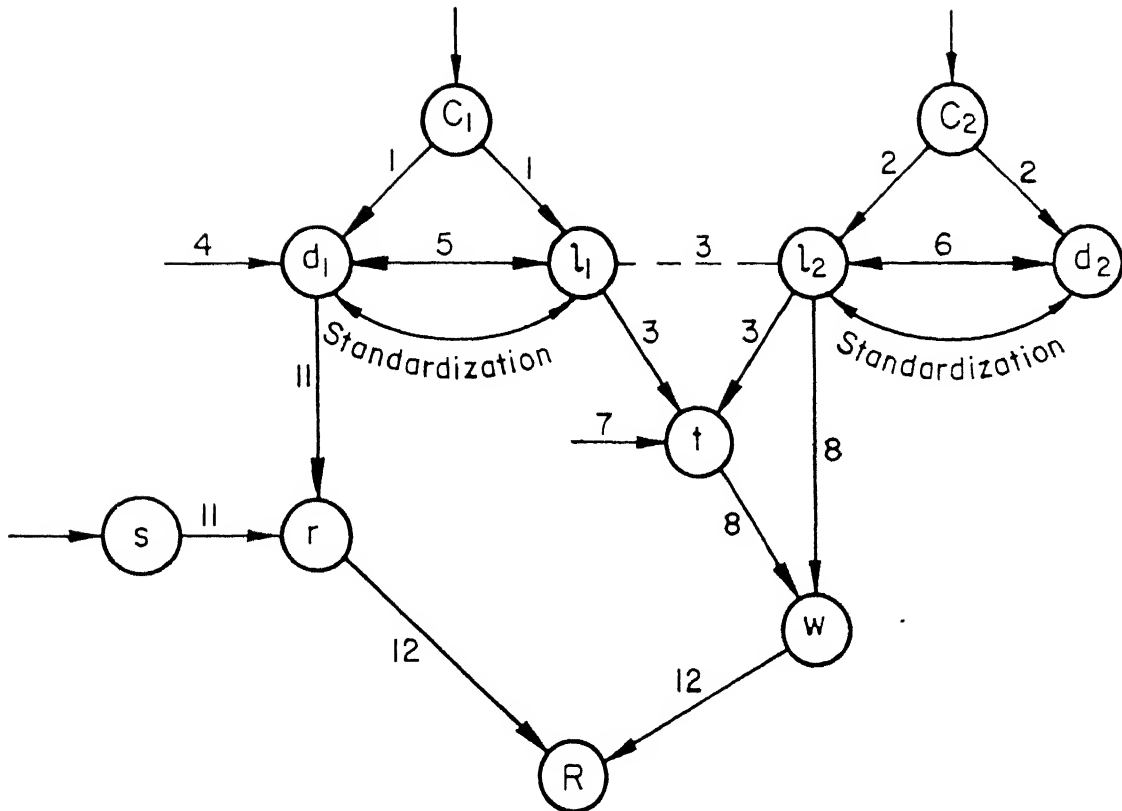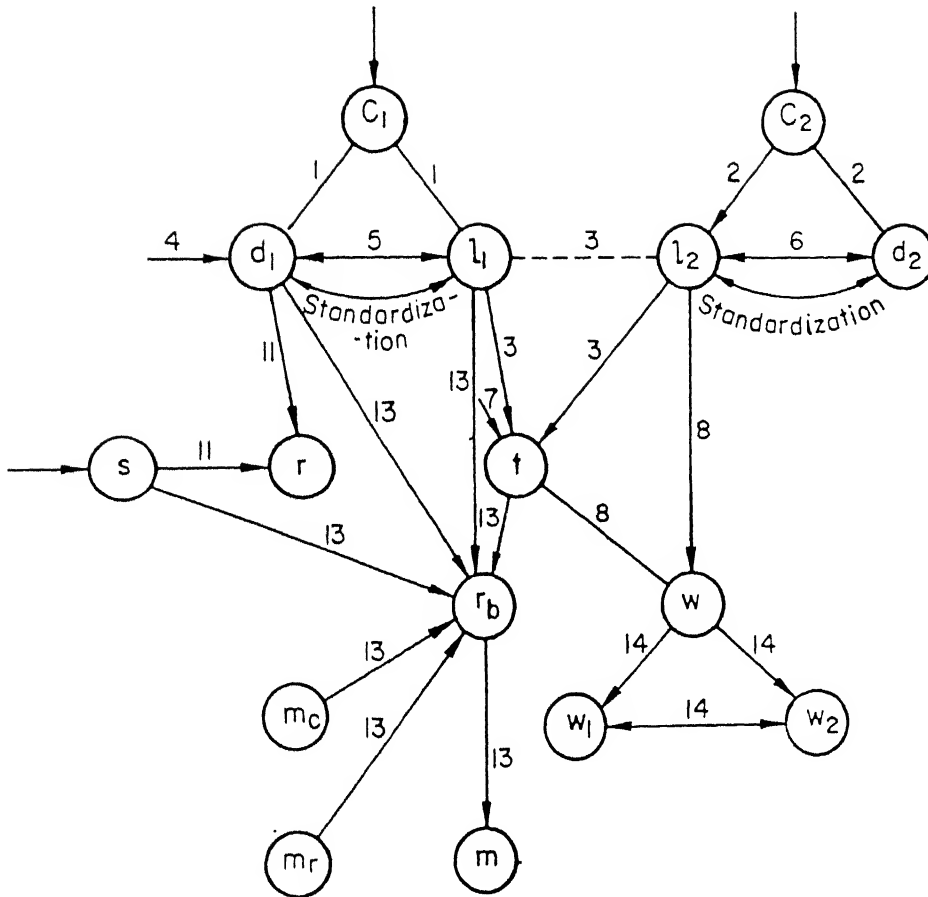
31

Figure 3.6: *PCD for circular crankshaft.*

Figure 3.7: *PCD for crankshaft similar to Bajaj scooter crankshaft.*

Chapter 4

Design process instantiation

At this stage in the design process we have completed the following steps.

1. Obtain a basic concept of geometry.

2. Expressed these geometries in a parametric model.

3. Derived the functional and other constraints interrelating the parameters.

The next step is to work through the constraints digraph (figure 3.5) and obtain a feasible set of values for the parameters. We call this process *design instantiation* since a specific instance of the parameter values are being determined.

The constraints represented by the parameter constraint digraph can be solved either manually or by a computer. Typically the manual process is sequential, and one proceeds from the most specific constraints (towards the top of the PCD) to the more general. In a computational optimisation approach, all the parameters are considered simultaneously subject to relevant constraints. We outline both the approaches for the crankshaft PCD (figure 3.5). The following assumptions are made for a two stroke scooter crankshaft.

We assume that the scooter runs 15 Km each day at on average rpm of 3000 rpm and average speed of 40 Kmph. The piston diameter is 45 mm and maximum gas pressure is 50 bar. Crankshaft is to be made of forged steel with allowable tensile strength 700 bar. The design life of the bearings are: crankshaft -5 years (ball bearing); crank-pin - 3 years (needle bearing). Weight of the piston, gudgeon pin, connecting-rod and crank-pin are 0.14 kg, 0.04 kg, 0.125 kg and 0.115 kg respectively.

4.1   Manual sequential solution

Any sequential solution may be subject to trial and error iterations since initial assumptions about parameter values may conflict with constraints considered later- hence we begin with the *most* specific constraints. In this case, we start with the bearing, for which the load and design life are specified. For the scooter assumptions above, we compute the maximum bearing load and

dynamic capacities for the desired bearing life. For the crank-pin bearing, from the several bearings that exceed this capacity, say we choose a bearing with $l_1 = 17\,mm$, $d_1 = 25\,mm$. In a similar way we select a main bearing with $l_2 = 17\,mm$, $d_2 = 25\,mm$. From constraint (3.3), we choose $t = 9.5\,mm$, and although this violates the empirical constraint (3.7), $9.9\,mm \leq t \leq 15\,mm$, the violation is small and empirical rules have a large tolerance. Constraints (3.5),(3.6) and (3.4) are already satisfied by the current parameters set. For the cheek geometry of figure 3.3, we have $w \geq 56\,mm$ from constraint (3.8); $r = 50\,mm$ from constraint (3.11), and $\theta = 127\,deg.$ from constraint (3.9). These values lead to $r_b = 48\,mm$ from constraint (3.10).

Circular cheek

We need to retain constraint(8) (According to which we must have material up to width w= 5.6 cm , which results from bending strength consideration at a section of cheek). Now look at figure 4.1. $DG = Stroke\,length/2 + d_1/2 = 30. + 24./2. = 42$ is the effective length as stress flow lines pass within this length only. To satisfy constraint(3.12) for a circular cross section, we must have a cross section of radius

$$R = GC = (42^2 + 28^2)^{1/2} = 50.5\,mm$$

This type of circular cross section are better than rectangular cross section of cheek in endurance strength [Taylor]. However, since this is not balanced for unbalance primary force so it will cause vibration, if used as such.
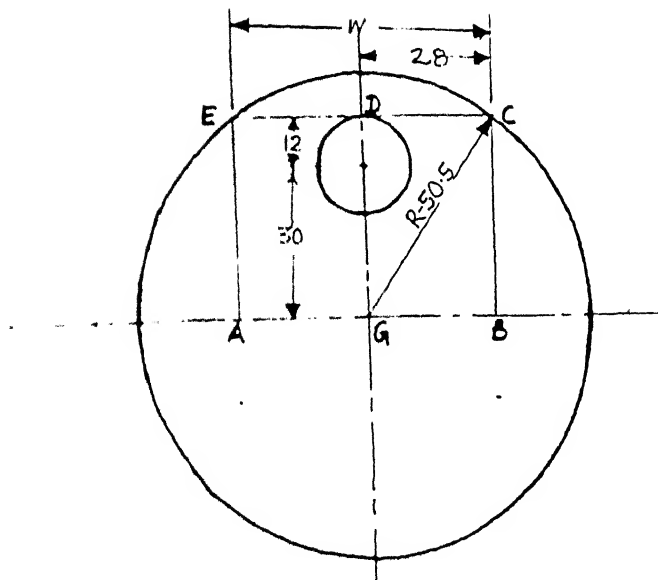
Figure 4.1: *Side view of circular cheek*

Check geometry variation: In figures 1.2 and 1.3 we saw two different types of cheek cross section, trapezoidal and circular. In practice the nominal geometry of figure 3.3 is rarely used due to sharp angles in the contour which may cause stress concentrations. This is yet another example where important design decisions precede the parametric design stage. Typically the section will either be trapezoidal as in figure 1.2 or will have gentle flaring as in figure 4.5a. In both these cases, the PCD will be different, since the design template and and parametrization are different. Even more different is the case of a circular contour like the concept under evolution in figure 1.3. Here counterweight balancing is achieved not by the outer shape but by creating holes in the top part of the cross section. In the PCD for a circular cheek, most constraints will be different due to new parametrization. Figure 4.6 shows an instantiated template.

On the whole the manual instantiation process outlined above tends to arrive at a single feasible solution, but this may not be optimal. However experienced designers often do very well in this approach because much of the design knowledge is not available in an explicit computational framework. Complete specification of the design constraints in a PCD will help improve this situation.

4.2   Computational search : Genetic Algorithm

In a computational search approach, we can seek a simultaneous instantiation for all the design parameters. This resembles the optimisation problem, where an objective function has to be minimized/maximized without violating any of the design constraints. However in this problem, like many others in design, no unique objective function can be stated; different designers may prefer different aspects to be emphasized. Also, the designer may wish to review a number of near-optimal solutions before making a decision. We feel that instead of overstating our achievements by claiming *optimality* we would prefer to consider our program not as an autonomous designer, but merely as a design assistant.

The basic question is: given a set of constraints, how does one find a feasible solution . Clearly some guidelines are needed. Also, we would like to find feasible solutions that may be equally good, but may differ significantly in

their composition.

Traditional optimization methods are limited by restrictive assumptions about the search space (continuity, existence of derivatives, unimodality etc.). Usually search proceeds in small steps. But newer methods such as genetic algorithm or statistical annealing are not limited by these assumptions and are computationally simple yet powerful in their search. Also this approach allows us to model discrete programming, such as the choice of bearings. Due to these reasons we adopted the genetic algorithm approach. Both mixed search (mixture of continuous and discrete variables) and continuous search were performed. Following decisions were made with respect to the problem.

- For mixed search design variables are (l_1, d_1), (l_2, d_2) and w. (l_1, d_1) and (l_2, d_2) were to be chosen from a finite set of choices in the design range and were represented by integer choices n_1 and n_2 respectively (Table 4.1). For continuous search l_1, d_1, l_2, d_2 are chosen as design variables. Parameters w, r_b are computed separately (Table 4.3).

- Chromosome composition:

    1. For mixed search chromosome length was 18. For each individual variable length was 6 (figure 4.2a).

    2. For continuous search chromosome length was 32. For each individual variable length was 8 (figure 4.2b).

- crossover probability was 1.0 and mutation probability was 0.01. Population size was 30. These values are typical for a large class of Genetic

algorithms.

- penalty function: Since many of the constraints are associated with a tolerance, it is permissible to to violate these constraints in a limited manner depending upon the nature of the associated tolerance. This is achieved by having a penalty function that increases linearly over the tolerance zone, and exponentially after that (figure 4.3).

- Fitness function: This is analogous to an objective function f. In this instance we chose weight or volume as this function. For mixed search this function was expressed as

$$F = \frac{\pi}{4}d_1^2\, l_1 + \frac{\pi}{2}d_2^2 l_2 + w\, \frac{(7 - l_1 - 2l_2)}{2}\, (38 + \frac{d_1}{2}) + \theta^{1/3}\, \frac{(7 - l_1 - 2l_2)}{4}\, (1.5\, w(38 + d_1/2)^2)^{2/3}$$

Based on this criteria, we come up with a number of feasible solutions that vary widely in their attributes; e.g. in the mixed search d_2 varies by 40 % between two solutions with fitness values 91 and 88 (Table 4.2). We believe that mixed search is more appropriate to most design problems since the standardization variables(here $n_1\ and\ n_2$) are usually discrete.
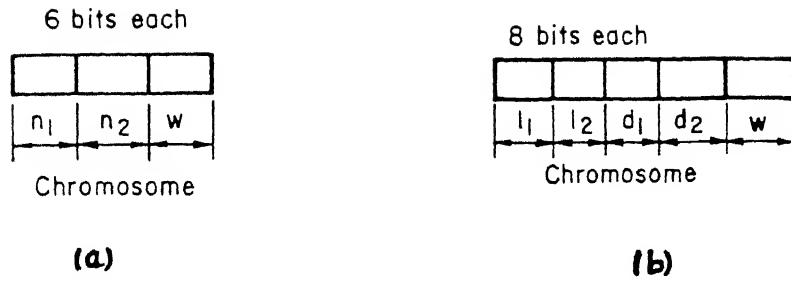
Figure 4.2: *chromosome structure. a*) Mixed search, *b*) Continuous search.

n_1	$l_1\,mm$	$d_1\,mm$	n_2	$l_2\,mm$	$d_2\,mm$
0	17	25	0	15	25
1	19	30	1	17	25
2	21	35	2	21	25
3	23	40	3	16	30
4	25	45	4	19	30
5	27	50	5	23	30
6	29	55	6	17	35
7	31	60	7	21	35
8	33	65	8	25	35

Table 4.1: *Table lookup for discrete search.* n_1- Variable for needle bearing, n_2- Variable for ball bearing.

$l_1\,mm$	$d_1\,mm$	$l_2\,mm$	$d_2\,mm$	$w\,mm$	fitness
17.0	25.0	17.0	25.0	60.0	76.3
17.0	25.0	17.0	35.0	57.9	90.82
19.0	30.0	15.0	25.0	59.2	87.36
17.0	25.0	15.0	25.0	52.5	88.3
19.0	30.0	15.0	25.0	51.7	80.9
19.0	30.0	15.0	25.0	50.3	79.7

Table 4.2: *Table shows feasible sets obtaind in Mixed search.* Results were obtained in 15 Generations

$l_1\,mm$	$d_1\,mm$	$l_2\,mm$	$d_2\,mm$	w mm	Fitness
10.5	28.7	18.5	22.1	47.4	73.3
19.3	26.0	13.8	10.3	38.5	57.18
19.5	29.0	13.8	13.2	39.0	62.5
12.5	26.7	18.5	15.7	54.4	66.59
10.6	27.6	18.5	11.8	47.78	61.91
19.5	26.5	13.8	11.9	39.0	57.89
10.5	26.7	18.5	11.8	47.4	60.92
19.1	26.7	13.8	10.3	38.08	56.98
19.1	25.3	13.7	10.3	37.4	54.93

Table 4.3: *Feasible sets from Continuous search.* Results were obtained in 49 generations. These sizes will not match actual bearing and some compromise will be needed.
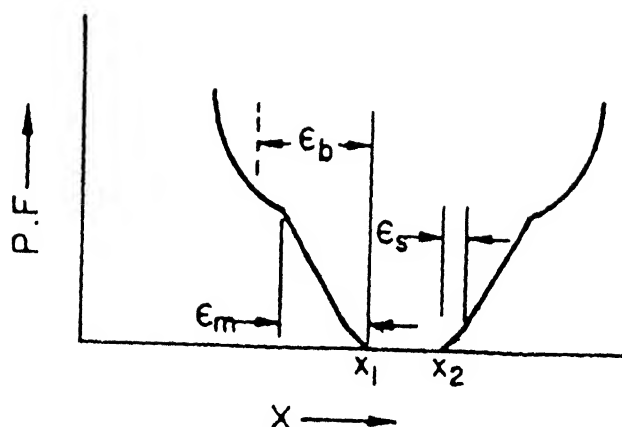
Figure 4.3: *Penalty function and tolerances*

4.3 Refinement:Adding/Modifying constraints

All designs are subjected to refinement. Typically this occurs at the conceptual stage. In this treatment of the two-stroke crankshaft the model(template) we have chosen is very elementary. The crankshaft may have different shapes depending upon functions they are to perform. The Parameter constraint digraph will change depending upon the functional requirements of the final shape(template, obtained after continuous refinement). The PCD which we discussed in section (3) is for the simple model (figure 3.3). One of the weaknesses of this model(template) are the stress concentrations at abruptly changing cross sections. One solution would be to introduce a smooth curved boundary for the cheeks, which would result in a different PCD which incorporates this new constraint on shape. By introducing this new constraint we get a model which resembles a moped crankshaft(figure 4.5a). It is known that the endurance strength of a crankshaft is maximum when cross section of cheek is circular. With this altered concept the PCD will have to be mod-

ified. This new circular shape model has been instantiated manually and the resulting geometry is shown in figure 4.5b. This model resembles the crankshaft of Vijay Super scooter of Scooters India Limited.

In some of the scooter crankshafts the crank's cheek has another function besides providing balancing weight, it also regulates the opening and closing of transfer port. For this we require extra material in upper periphery of the crank shaft. Further we already have the constraint that crankshaft should be statically balanced, which necessitates the removal of a circular disc from the cheek. Thus we have only that much amount of peripheral material which is required to regulate the opening and closing of the transfer port and the balancing is not disturbed much. Since removing a disc will weaken the cheek in bending so material is added on other side of the crank arm. In this case this new functional constraint on crankshaft (that it should regulate the transfer port) affects both ,the constraints at stage(a) and PCD at stage (b) (refer chapter 2.5). We have instantiated this model also (figure 4.6) and it is very similar to a Bajaj scooter crankshaft. This shows that from PCD by suitable instantiation we can reach close to different types of designs, as used in practice.

4.4   Shape-function relationship:

A basic question is how function affects the shape. In most mechanical designs the shape (basic template) is derived by constraints/functions sat-
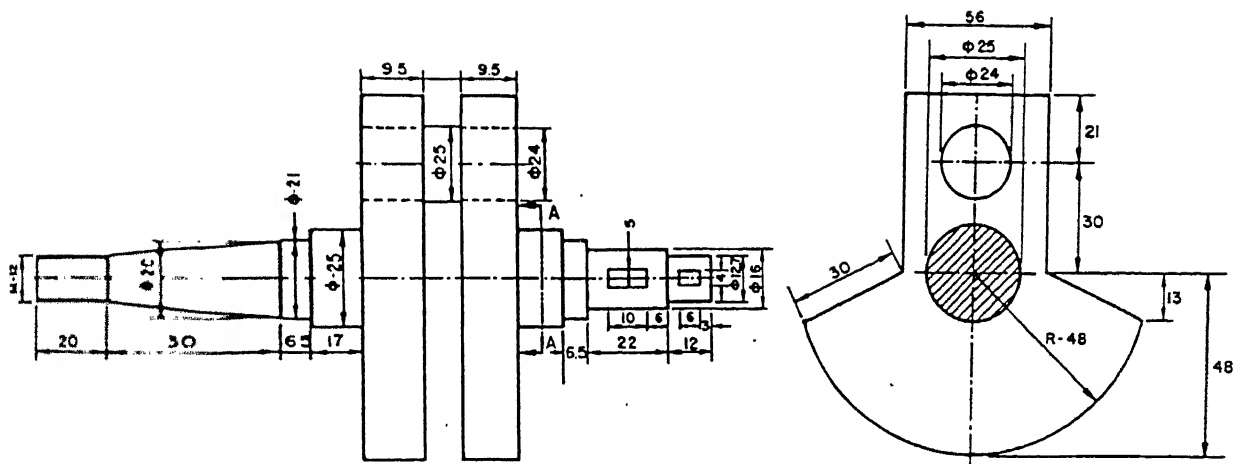
Figure 4.4: *Instantiated drawing of crankshaft, showing values of parameters.*
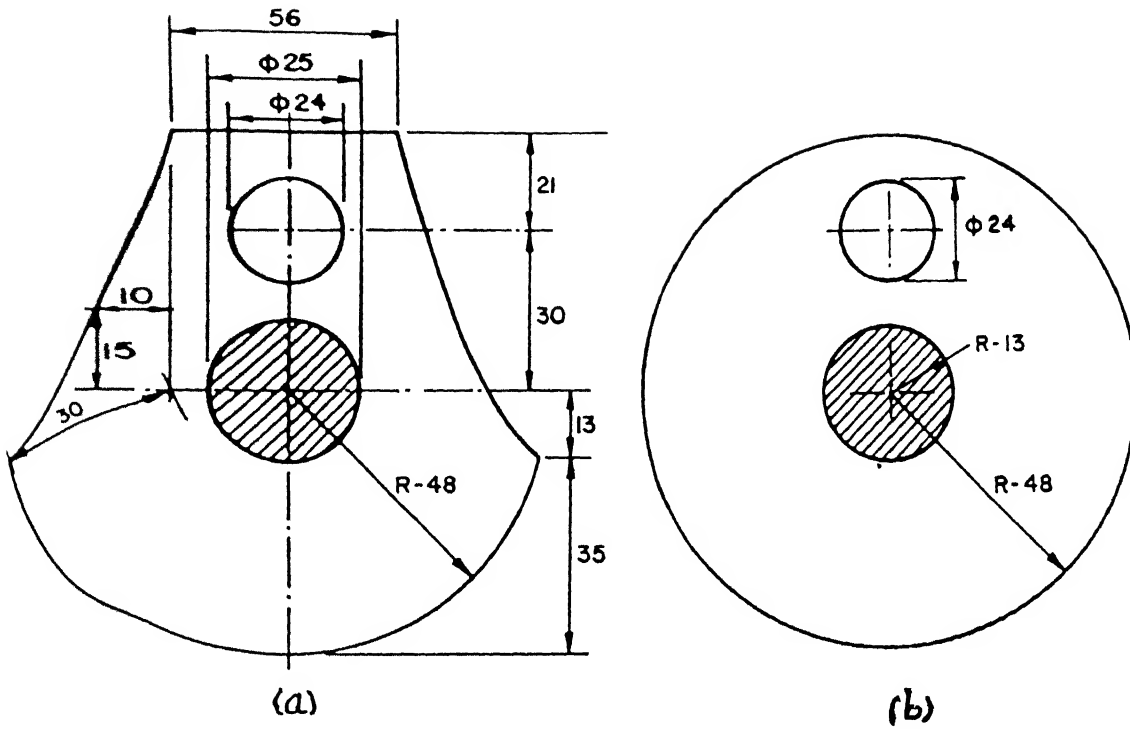The magneto and clutch dimensions are external.

Figure 4.5: *Two alternative designs for the cheek.* (a) A curved cheek to reduce stress concentration at the corners. Similar designs are used in lower power mopeds. (b) Circular cheek. This design is similar to that used in the Vijay super scooter.
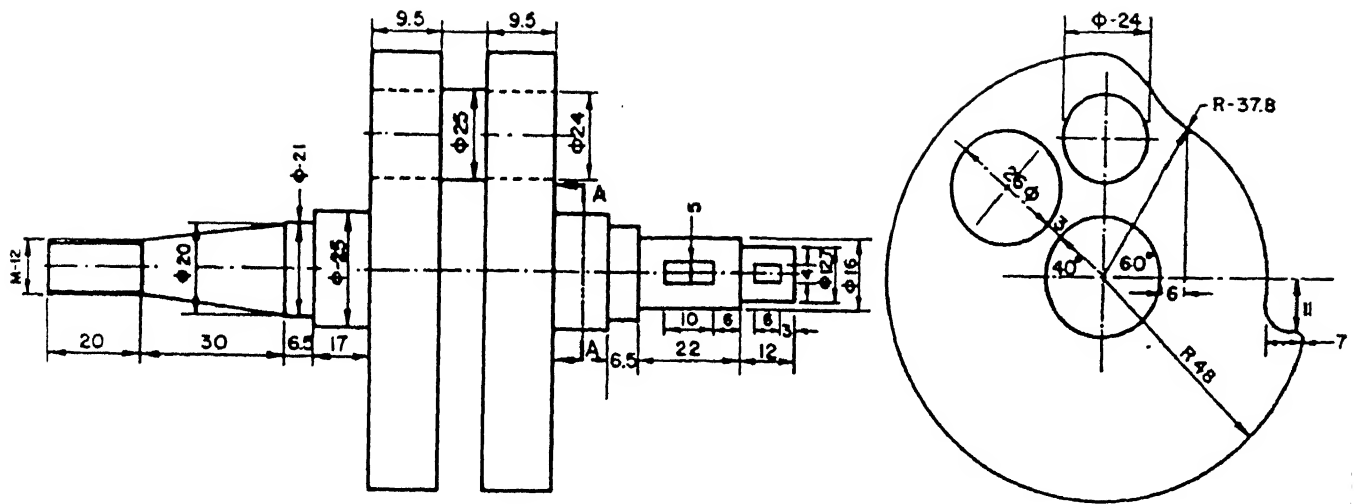
Figure 4.6: *Crankshaft similar to Bajaj scooter.* In the side view of this crank periphery of crank need to regulate opening and closing of transfer port

isfaction process. The basic shape of the crankshaft is designed keeping in mind its function to transmit motion of the connecting rod to rotary motion of the crankshaft. Some functions directly affect the shape e.g. function of the cheek to open and close the transfer port requires extra periphery of material (figure 4.6). In the same instance Balancing function and material saving requirement require a circular disc to be removed to the left of the crank arm. Further strength constraint requires adding material to the right of the crank arm (figure 4.6). In some of the products aesthetic requirements dominate the shape (as in a car body).

If the basic function (changing motion of connecting rod to rotary motion of main shaft) remains same then for modified shapes we can get modified PCD from previous PCD. In the current example we can go from PCD of moped crankshaft to 1) PCD of Circular crankshaft (figure 3.6). 2) PCD of crankshaft of figure 4.6 (figure 3.7).

Chapter 5

Summary and conclusion:

This thesis deals with visualisation and gradual refinement at conceptual stage, i. e. when the shape of the component is not completely defined. We use a Qualitative medial axis transform and generalized cylinders as tools for representing this initial vague shape. At this stage the shape represents a class instead of a *particular* shape. A number of parameters of the shape are open, and by assigning different values to these parameters, one obtains different instances for the same design solution.

While parametric designs have been modeled in the literature (e.g. geometric parameter constraints, [Light and Gossard]), most of these have been equality constraints where the model is much more restrictive. There has been little attempt to link the constraint with functional aspects so that during later editing, one can immediately realize the inter-relations between the various functions involved.

Thus this thesis addresses two issues, both of which are novel in their scope and content. On the one hand our model uses Hybrid Qualitative Spatial Reasoning to represent a class of objects as opposed to a single object, and on the other hand, it provides a graph structure for the constraints, linking them to functions. Since the design of a product goes through many revisions during its life cycle, the capability to organize the functional needs at design time is a crucial issue.

One of the problems that remain is that the constraints that were defined during the evolution of the parametric model are not within the scope of the PCD. In some sense, this part of the design - the creation of the template, so to say, is the more creative, the more varied task during the act of design. It is said that 80% of the design value is locked up in the initial design template. In this work, we have merely outlined a tool for capturing the process of this evolution; we are still far from evaluating the possibilities and suggesting solutions ourselves. However, this is clearly a large step from what CAD systems are currently capable of.

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